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THE HYBRID BOUNDARY ELEMENT METHOD APPLIED TO LINEAR FRACTURE MECHANICS

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Abstract. *The hybrid boundary element method, a variational formulation introduced in 1987 by the first author, has proven to be a powerful tool for dealing with a great variety of problems of potential, elastostatics and elastodynamics. One of the most recent endeavors of the research team at PUC-Rio is the application of the method to problems of the linear elastic fracture mechanics. In the M.Sc. dissertation of the second author, several possible formulations were introduced and discussed. Aim of the present paper is the outline of the main conceptual results of this dissertation, which are to be further elaborated in the frame of a Ph.D. work. The mechanical consistency of the hybrid boundary element method enables the adequate mathematical description of the stress field related to a cracked continuum. The effects of opening and sliding of the crack surfaces are considered by means of a hypersingular stress function, which is dealt with adequately in the frame of the variational formulation. Moreover, it is shown that the stress series proposed by Williams for the local field around a crack tip may be considered as a Green's function. As a consequence, the stress intensity factors corresponding to both modes I and II are directly obtained as primary unknowns of the problem. Some numerical examples are presented in order to validate the formulations proposed.*

Keywords: *Fracture mechanics, Hybrid boundary element method, Stress intensity factor.*

1. A BRIEF OUTLINE OF THE HYBRID BOUNDARY ELEMENT METHOD

The hybrid boundary element method is based on the Hellinger-Reissner potential. It has been developed by Dumont (1987), who generalized Pian's ideas for finite elements by considering the stress field in the domain as a series of fundamental, singular solutions, thus arriving at a boundary integral formulation for an elastic body of arbitrary shape.

Stresses in the domain. One is looking for an adequate approximation of the stress field that satisfies equilibrium in the domain, for applied body forces b_i:

$$\sigma_{ij},_{j} + b_i = 0 \quad \text{in } \Omega \tag{1}$$

In this and in the following equations, the indices i, j may assume values 1, 2 or 3, as they refer to the coordinate directions x, y or z, respectively, for a general three-dimensional analysis. Later on, one shall particularize the application to two-dimensional problems. Sum is indicated by repeated indices.

A convenient approximate solution of the partial differential equation (1) may be formulated in terms of a superposition of two types of fields:

$$\sigma_{ij} = \sigma_{ij}^{*} + \sigma_{ij}^{b} \tag{2}$$

in which σ_{ij}^{b} is an arbitrary particular solution of eq. (1):

$$\sigma_{ij}^{b} + b_i = 0 \tag{3}$$

and σ_{ij}^{*} is expressed as a sum of fundamental solutions

$$\sigma_{ij}^{*} = \sigma_{ijm}^{*} p_m^{*} \tag{4}$$

for singular force parameters p_m^{*} applied along the boundary Γ, but just outside the domain. The subscript m indicates a given degree of freedom, characterizing not only a point along Γ but also a coordinate direction. Since σ_{ij}^{*} corresponds to a fundamental solution, the homogeneous solution of eq. (1) is identically satisfied. In fact,

$$\sigma_{ij}^{*},_{j} = \sigma_{ijm}^{*},_{j}\, p_m^{*} = 0 \quad \text{in } \Omega \tag{5}$$

except in a vicinity Ω_0 of the point of application of the singular force p_m^{*}, where

$$\int_{\Omega_0} \sigma_{ijm}^{*},_{j}\, d\Omega = \begin{cases} 1 & \text{if } i \text{ and } m \text{ refer to the same degree of freedom} \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

From the stresses in eq. (4) one derives the traction forces along the boundary Γ as

$$t_i^{*} = p_{im}^{*} p_m^{*} \tag{7}$$

Displacements along the boundary. Along the boundary, the displacements are piecewise approximated by polynomials

$$u_i = u_{in} d_n \tag{8}$$

in such a way that boundary conditions along Γ_u are identically satisfied:

$$u_i = \bar{u}_i \quad \text{in } \Gamma_u \tag{9}$$

in which $\bar{u}_i$ are prescribed displacements.

For prescribed body forces b_i in Ω, displacements $\bar{u}_i$ in Γ_u and traction forces $\bar{t}_i$ along Γ_σ, the problem may be completely described once the singular force parameters p_m^{*}, for stresses in the domain, and the nodal displacement parameters d_n, for displacements along Γ, have been determined.

The variational statement. The problem, as proposed above, may be formulated in terms of the variational statement that the Hellinger-Reissner potential

$$-\Pi_R = \int_\Omega \left[\frac{1}{2}\sigma_{ij}\varepsilon_{ij} + \left(\sigma_{ij,j} + b_i\right)u_i \right] d\Omega - \int_\Gamma \sigma_{ij}\eta_j u_i d\Gamma + \int_{\Gamma_\sigma} \bar{t}_i u_i d\Gamma \tag{10}$$

is stationary. Given the description of stresses, forces and displacements proposed, one may rewrite the stationariness statement in matrix notation:

$$-\delta\Pi_R = \delta\mathbf{p}^{*\mathrm{T}}\left[\mathbf{F}\mathbf{p}^* + \mathbf{b}^{\mathrm{b}} - \mathbf{H}\mathbf{d}\right] + \delta\mathbf{d}^{\mathrm{T}}\left[\mathbf{p} - \mathbf{t}^{\mathrm{b}} - \mathbf{H}^{\mathrm{T}}\mathbf{p}^*\right] = 0 \tag{11}$$

in which the flexibility matrix $\mathbf{F} \equiv F_{mn}$**,** the cinematic transformation matrix $\mathbf{H} \equiv H_{mn}$ and the vector of equivalent nodal displacements $\mathbf{b}^{\mathrm{b}} \equiv b_m^b$ are defined, in a compact notation, as

$$\left[\mathbf{F} \quad \mathbf{H} \quad \mathbf{b}^{\mathrm{b}}\right] = \int_\Gamma \left\{p_{im}^*\right\}\left\langle u_{in}^* \quad u_{in} \quad u_{in}^b\right\rangle d\Gamma - \int_\Omega \left\{\sigma_{ijm,j}^*\right\}\left\langle u_{in}^* \quad u_{in} \quad u_{in}^b\right\rangle d\Omega \tag{12}$$

Similarly, the vector of equivalent nodal forces $\mathbf{t}^{\mathrm{b}} \equiv t_m^b$ and the vector $\mathbf{p} \equiv p_m$, which is in part a set of nodal forces equivalent to known surface forces t_i along part Γ_σ of the boundary, and in part a set of unknowns corresponding to reaction forces along the complementary boundary segment Γ_u, are expressed as

$$\mathbf{t}^{\mathrm{b}} = \int_\Gamma \sigma_{ij}^b \eta_j u_{im} d\Gamma \quad \text{and} \quad \mathbf{p} = \int_{\Gamma_\sigma} u_{im} t_i d\Gamma \tag{13}$$

For arbitrary variations $\delta\mathbf{p}^*$ and $\delta\mathbf{d}$, two sets of equations originate from the variational principle:

$$\begin{aligned} \mathbf{F}\mathbf{p}^* &= \mathbf{H}\mathbf{d} - \mathbf{b}^{\mathrm{b}} \\ \mathbf{H}^{\mathrm{T}}\mathbf{p}^* &= \mathbf{p} - \mathbf{t}^{\mathrm{b}} \end{aligned} \tag{14}$$

For a finite domain, the matrix **H** is singular by construction, since the forces are void for any set of rigid body displacements. Then, defining an orthogonal basis **W** of rigid body nodal displacements, one must have from the first of eqs. (14):

$$\mathbf{H}\mathbf{W} = \mathbf{0} \tag{15}$$

As a consequence, there also exists an orthogonal basis **V** such that

$$\mathbf{H}^{\mathrm{T}}\mathbf{V} = \mathbf{0} \tag{16}$$

Moreover, it may be verified that, since in the second of eqs. (14) the equivalent nodal forces must be in equilibrium,

$$\mathbf{W}^{\mathrm{T}}\left(\mathbf{p} - \mathbf{t}^{\mathrm{b}}\right) = \mathbf{0} \tag{17}$$

Then, one must have, for physical consistency,

$$\mathbf{V}^{\mathrm{T}}\mathbf{p}^* = \mathbf{0} \tag{18}$$

from which follows, in the first of eqs. (14), that necessarily

$$\mathbf{F}\mathbf{V} = \mathbf{0} \tag{19}$$

This equation is the key for the evaluation of the elements about the main diagonal of the matrix **F**, which cannot be directly obtained by integration.

Considering the spectral properties given by eqs. (18) and (19), one may solve the first of eqs. (14) for $\mathbf{p}^*$, in terms of generalized inverses (Ben-Israel and Greville, 1980) and introduce its expression into the second of eqs. (14), thus arriving at the relation

$$\mathbf{H}^{\mathrm{T}}\left(\mathbf{F}+\mathbf{V}\mathbf{V}^{\mathbf{T}}\right)^{-1}\mathbf{H}\ \mathbf{d}=\mathbf{p}-\mathbf{t}^{\mathrm{b}}+\mathbf{H}^{\mathrm{T}}\left(\mathbf{F}+\mathbf{V}\mathbf{V}^{\mathbf{T}}\right)^{-1}\mathbf{b}^{\mathrm{b}} \tag{20}$$

in which

$$\mathbf{H}^{\mathrm{T}}\left(\mathbf{F}+\mathbf{V}\mathbf{V}^{\mathrm{T}}\right)^{-1}\mathbf{H}=\mathbf{K} \tag{21}$$

is a symmetric, positive semi-definite stiffness matrix. Owing to the spectral property of **H** given by eq. (15), this stiffness matrix is by construction orthogonal to rigid body displacements.

For the sake of brevity, one has to content oneself with this short description of the hybrid boundary element method. Interested readers are referred to some of the articles written by the first author in the last decade.

2. SOME FACTS FROM THE LINEAR ELASTIC FRACTURE MECHANICS

Aim of this outline is to introduce some definitions and aspects of the linear elastic fracture mechanics, which are relevant for the subject of this paper. Readers interested in being initiated into the theory of fracture mechanics are referred to Anderson (1995), for instance, who also outlines the development of this new discipline since the pioneering work done by Inglis (1913).

A crack in some region of an elastic body brings along a discontinuity of the stress field with abrupt increase of the stresses around the crack tip. It is the structural analyst's task to investigate whether the cracked elastic body is still able to bear the applied loading. For this sake, Griffith (1920) proposed a criterion based on a "global energy balance: for fracture to occur, the energy stored in the structure must be sufficient to overcome the surface energy of the material" (Anderson, 1995). This proposition was later expressed more conveniently by Irwin (1948) in terms of an energy release rate, $\mathcal{G}$, for linear elastic materials. Finally, Rice (1968) extended the concept for nonlinear materials, in terms of the J contour integral:

$$\mathrm{J}=\frac{d\Pi}{d\mathrm{A}} \tag{22}$$

where Π is the potential energy and A is the crack area. As Rice (1968) demonstrated, the J integral is contour independent. This integral is a fracture characterizing parameter for nonlinear materials. In case of linear materials, since J coincides with $\mathcal{G}$, it may be brought in connection to the stress intensity factors of a crack.

For cracked configurations subjected to external forces, several authors derived closed-form expressions for the stresses in the body. In terms of a polar coordinate system with the origin at the crack tip, it can be shown that the stress field in any linear elastic cracked body is given by

$$\sigma_{ij}=\sum_{k=1}^{3}\frac{\mathrm{K}_k}{\sqrt{2\pi r}}f_{ijk}(\theta)+\sum_{k=1}^{3}\sum_{l=0}^{\infty}A_{lk}r^{l/2}g^{ijlk}(\theta) \tag{23}$$

in which $\mathrm{K_I}$, $\mathrm{K_{II}}$ and $\mathrm{K_{III}}$ are defined as the stress intensity factors corresponding to the modes of loading I (crack opening), II (in-plane shear of the crack) and III (out-of-plane shear), respectively. For plane stress problems, expression above is known as William's series expansion. It is applicable to different local crack configurations (Williams, 1957).

Evaluating either integral J around a crack tip or the corresponding stress intensity factors is a means of assessing the stability of a cracked body. Then, one must demonstrate the adequacy of the hybrid boundary element method for the determination of either type of quantities. A third type of measuring the fracture toughness is the crack tip opening displacement, as proposed by Wells (1954). This is a preferred procedure in many boundary element implementations, since this type of analysis is computationally less expensive than the evaluation of the integral J. However, the crack tip opening displacement shall not be considered in this paper.

3. LINEAR ELASTIC FRACTURE ANALYSIS USING SUB-REGIONS

By means of a first numerical example, one is aiming to demonstrate the capability of the hybrid boundary element method in representing the stress field around a crack tip. For this sake, consider the analysis of a plate with a center crack and stretched uniaxially, as illustrated in the inset *b* of Fig. 1. Figure 1a represents one quarter of the plate, discretized with 42 linear boundary elements. The load is applied at the upper edge. Both left and lower edges are fixed against normal displacement, with exception of the boundary corresponding to the cracked surface, which, as well as the right edge, is completely free to displace. Since in this idealization only one of the crack surfaces is discretized, no topological difficulty arises. The stress field discontinuity around crack tip is due to the abrupt change of boundary conditions. Then a local mesh refinement is required, as detailed in Fig. 1c.

Owing to the occurrence of mixed boundary conditions, the problem has to be analyzed using both sets of equations (14), as given by eq. (20). After carrying out the analysis, half the value of the integral J can be obtained numerically along the semi-contour indicated in Fig. 1d, which is a half square with edges equal to $2*10^{-5}$ units. Since the region corresponds to high stress gradients, a highly accurate numerical evaluation is required. In the example, 15 Gaussian points were used for each one of the contour segments indicated in the figure. The numerical result obtained was J = 0.01215, corresponding to a mode I stress intensity factor K_I = 1.77684. For comparison, the analytical value given by Anderson (1995) is K_I = 1.77713 (a 0.016% error).

In the hybrid boundary element method, the evaluation of the integral J is by far less computationally expensive than in the conventional formulation, since the internal results along the contour are given directly by means of eqs. (2) and (4) in terms of $\mathbf{p}^*$.

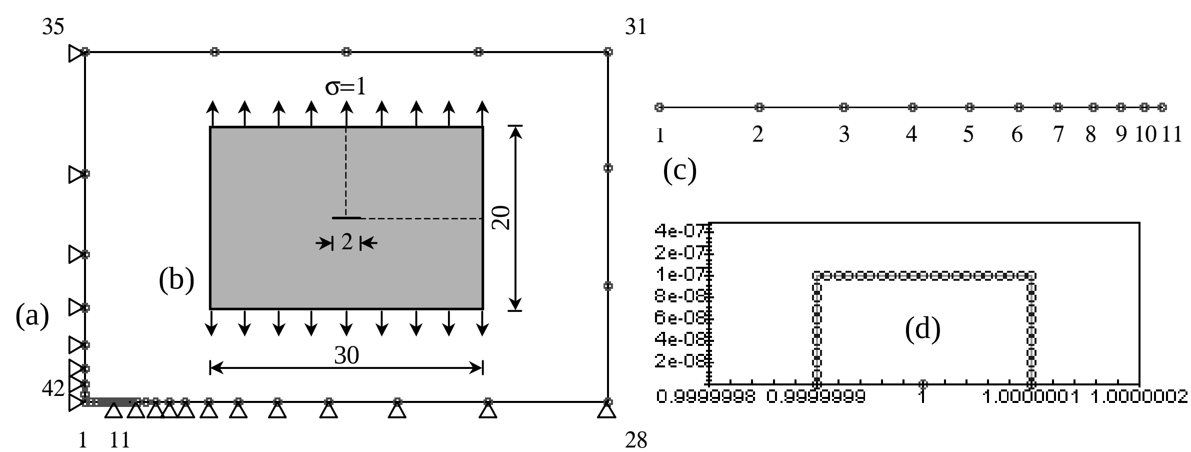

Figure 1 – a) Boundary element discretization of one quarter of a rectangular plate with a center crack loaded uniaxially (inset b); c) detail of mesh refinement along the crack face; d) half square contour around crack tip (node 11).

4. LINEAR ELASTIC FRACTURE ANALYSIS USING HYPERSINGULAR FUNCTIONS

Before any further consideration, it is important to point out that, in case of Neumann-type boundary conditions, the hybrid boundary element method offers a simplification that cannot be matched by any other numerical discretization method, namely the fact that only the second of eqs. (14) is needed in the analysis, since the vector ($\mathbf{p} - \mathbf{t}^b$) of equivalent nodal loads is completely known and only the vector $\mathbf{p}^*$ of singular forces is required for the representation of the stress state in the interior of the elastic body.

In spite of the stimulating results obtained using sub-regions, as outlined in the previous section, the authors wanted to investigate the possibility of discretizing a whole structure, which requires overcoming the topological difficulty of representing in a single model both faces of a crack.

This challenge has already been dealt with successfully in the frame of the conventional boundary element method. As a consequence, there should be no fundamental obstacle in implementing a well tested procedure. In the conventional boundary element method, hypersingular fundamental solutions are related to nodes along the crack surface, whereas the usual Kelvin's fundamental solution is applied to the crack tip. This recipe also should work with the hybrid boundary element method.

The stress field around contiguous faces of a crack suffers a discontinuity given by the tendency of relative opening and sliding of the surfaces (the possibility of friction shall be completely ignored herewith). Then, a hypersingular fundamental solution should be appropriate for the representation of such discontinuity. However, differently from what some researchers have already proposed in the frame of the conventional boundary element method, the authors preferred to simulate the stress discontinuity along the crack surface in terms of direct differences of singular forces. [see Guimarães and Telles (1994), for instance; a literature review would be too long] Figures 2a and 2b represent a point of the crack surface, for a two-dimensional analysis, to which two couples of singular forces are applied, as an attempt of simulating the neighboring stress field.

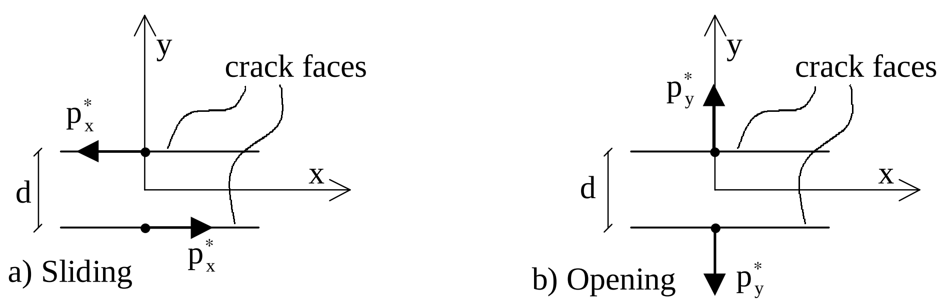

Figure 2 – Hypersingular fundamental solutions given as differences of Kelvin's fundamental solutions for the representation of sliding and opening at a point of the crack face.

For the local Cartesian coordinates system, the couples of forces are applied in such a way that $\lim_{d\to 0}\frac{\Delta p^*}{d} = 1$. Then, for displacements u^*_{ik} and stresses σ^*_{ijk} of Kelvin's fundamental solution, corresponding to unitary singular forces applied in the direction *k*, for the local Cartesian system of Fig. 2, the hypersingular displacements and stresses are simply

$$u^h_{ik} = \frac{\partial u^*_{ik}}{\partial y}, \quad \sigma^h_{ijk} = \frac{\partial \sigma^*_{ijk}}{\partial y} \tag{24}$$

For numerical implementation, these expressions must be rewritten in global Cartesian coordinates system (Lopes, 1998).

This approach in terms of hypersingular fundamental solutions does not differ essentially from previous accomplishments in the conventional boundary element method. However, there is a remarkable difference in terms of describing stresses and displacements around the crack tip. In the conventional boundary element method, a boundary integral statement – Somigliana's identity – gives displacements in the interior of the domain by using a fundamental solution that plays the role of a weighting function. Then, it is possible to consider Kelvin's fundamental solution applied directly at the crack tip, combined with the hypersingular solution along the crack face.

In the hybrid boundary element method, on the other hand, Kelvin's fundamental solution is itself the actual result one is looking for, in terms of the singular load parameters $\mathbf{p}^*$. Then, if one considers that the fundamental solution is applied at the crack tip, one is tacitly assuming that the stress singularity increases with $1/r$ and not with $1/\sqrt{r}$, as obtained in the theory of fracture mechanics, according to eq. (23). Such an implementation, as tested by Lopes (1998), yields the obvious result of a zero load parameter p_m^* for the crack tip.

In the sequence of investigation, the authors reasoned that maybe the hypersingular functions alone, applied along the crack face and with no singular loading at the crack tip, are sufficient to describe the stress field around a crack. The same plate of Fig. 1 was investigated again, as a whole structure, with Kelvin's fundamental solution applied at points along the boundary and the hypersingular fundamental solution applied at points along the crack face. Twenty evenly spaced linear elements were considered along the boundary. The same number of elements was used along the crack face. Since plain Neumann boundary condition is involved in this problem, only the second of eqs. (14) is required in the analysis. The numerical evaluation of the matrix $\mathbf{H}$ is straightforward, for the complementary terms involving the hypersingular fundamental solution, if one considers that conceptually only the finite part of the eventual singular integrals is required. Guimarães and Telles (1994) elucidate this fact mathematically. However, differently from the example of Fig. 1, now one is not able to obtain the stress intensity factor through the integral J, since this integral is simply void. This assertion becomes immediately obvious when one considers that no fundamental solution is applied at the crack tip and therefore no actual discontinuity has been provoked.

On the other hand, one may demonstrate numerically that, at least for this type of discretization, the stress field around the crack, although continuous, presents the same tendency of the actual stress field. Figure 3 displays the results of σ_{yy} along the middle section of the plate, for three different sizes of the central crack. For comparison, the theoretical stress field of eq. (23), as proposed by Williams (1953), is evaluated by adjusting the coefficients K_k and A_{mk} (for mode I alone) using least squares. The excellent agreement of results may be verified visually. The values of K_I, as obtained by this asymptotic approach, are given in Table 1.

Table 1 – Mode I stress intensity factors for the plate of Fig. 3

Relation a/W	Coordinate of crack tip	Interval used for least squares	K_I Numerical	K_I analytical
1/15	(16, 15)	(16.135, 15) – 22.135, 15)	1.77924	1.77713
1/10	(11, 10)	(11.135, 10) – (17.135, 10)	1.78874	1.78303
1/5	(6, 5)	(6.136, 5) – (8.735, 5)	1.80216	1.81584

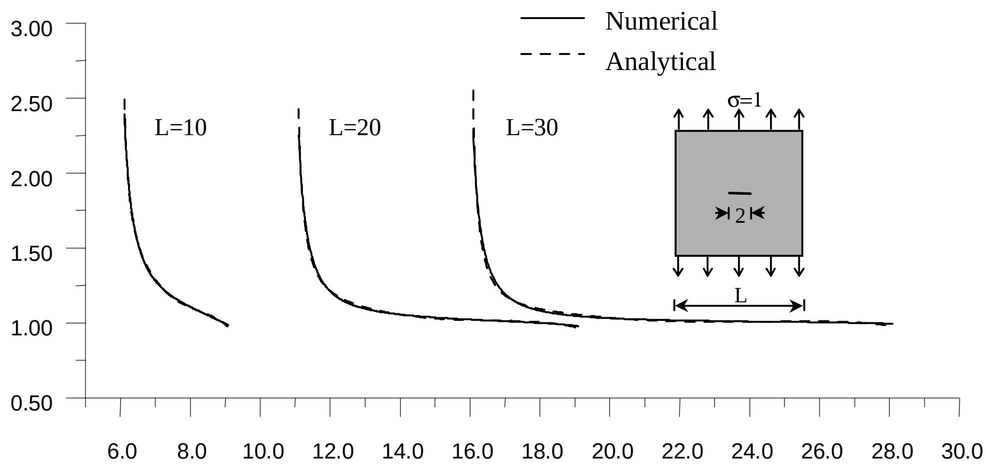

Figure 3 – Stress gradients along the symmetry line of a rectangular plate with a center crack for three different crack/width ratios. Dashed curves are Williams' series with coefficients evaluated in order that the numerical result fits best. The stress intensity factor K_I is one of these coefficients.

5. LINEAR ELASTIC FRACTURE ANALYSIS USING WILLIAM'S SERIES AS A GREEN'S FUNCTION

Consider a plate (of arbitrary shape) with an edge crack, as illustrated in Fig. 4a, submitted to a self-equilibrated, but otherwise arbitrary loading throughout the boundary. Body forces are absent, although they also might be acting. The crack face may be either loaded or unloaded. For this particular case of one edge crack, the stress field in the plate may be expressed by (Lopes, 1998)

$$\sigma_{ij} = \sigma^*_{ijm} p^*_m + \sigma^w_{ijl} p^w_l \tag{25}$$

in which one considers a stress field $\sigma^*_{ijm} p^*_m$ given as a superposition of Kelvin's fundamental solutions, for singular forces p^*_m applied along the boundary as if there was no crack, superposed with a stress field $\sigma^w_{ijl} p^w_l$ given by William's series expansion. This stress field corresponds exactly to eq. (23), for plane stress problems, with as many terms as desired for accuracy, including the singular term. It is a Green's function for the single edge crack. Compare eq. (25) with eqs. (2) and (4).

Moreover, the displacement field along the boundary is assumed as

$$u_i = u_{im} d_m + u^w_{il} p^w_l \tag{26}$$

which is a more general expression than eq. (8). Then, the matrix equilibrium equation given in eq. (14) is rewritten as

$$\begin{bmatrix} \mathbf{H}^{**} & \mathbf{H}^{*w} \\ \mathbf{H}^{w*} & \mathbf{H}^{ww} \end{bmatrix} \begin{Bmatrix} \mathbf{p}^* \\ \mathbf{p}^w \end{Bmatrix} = \begin{Bmatrix} \mathbf{p} \\ \mathbf{p}^w_{eq} \end{Bmatrix} \tag{27}$$

in which

$$\begin{bmatrix} \mathbf{H}^{**} & \mathbf{H}^{*w} \\ \mathbf{H}^{w*} & \mathbf{H}^{ww} \end{bmatrix} = \int_\Gamma \begin{Bmatrix} p^*_{im} \\ p^w_{il} \end{Bmatrix} \langle u_{in} \quad u^w_{ik} \rangle d\Gamma - \int_\Omega \begin{Bmatrix} \sigma^*_{ijm},_j \\ 0 \end{Bmatrix} \langle u_{in} \quad u^w_{ik} \rangle d\Omega \tag{28}$$

and

$$\begin{Bmatrix} \mathbf{p} \\ \mathbf{p}_{eq}^{w} \end{Bmatrix} = \int_{\Gamma_\sigma \equiv \Gamma} \begin{Bmatrix} u_{im} \\ u_{il}^{w} \end{Bmatrix} t_i \, d\Gamma \qquad (29)$$

Note that, in eq. (28), $\mathbf{H}^{**} \equiv \mathbf{H}$, according to eq. (12). Moreover, the vector of equivalent nodal forces **p** had already been defined in eq. (13). The evaluation of the remaining integrals in eqs. (28) and (29) is straightforward, since William's series involves no new singularities.

In a last numerical example, equation (27) is applied to the solution of the edge crack problem illustrated in Fig. 4a. Two boundary discretizations are implemented, as shown in Figs. 4b and 4c. Subscripts m and n in the equations above refer to the nodal degrees of freedom along the boundary. Subscripts l and k refer to odd terms of William's series, in such a way that only fractional exponents of the radius r are considered, as given in eq. (23), since the polynomial part of this series is already represented by Kelvin's fundamental solution applied along the boundary. Note that the crack face has not been discretized.

Calculated values of stress intensity factors are given in Table 2. The results are adequate, given the coarse mesh discretization. Note that this mathematical model does not simulate exactly freeloaded edges, since stresses due to Kelvin's fundamental solution are acting in the crack faces. However, the stress field around the crack tip is adequately simulated by William's series, which compensates the model's fault. This simple model fails completely in simulating multiple cracks, since William's series is valid for just one crack tip.

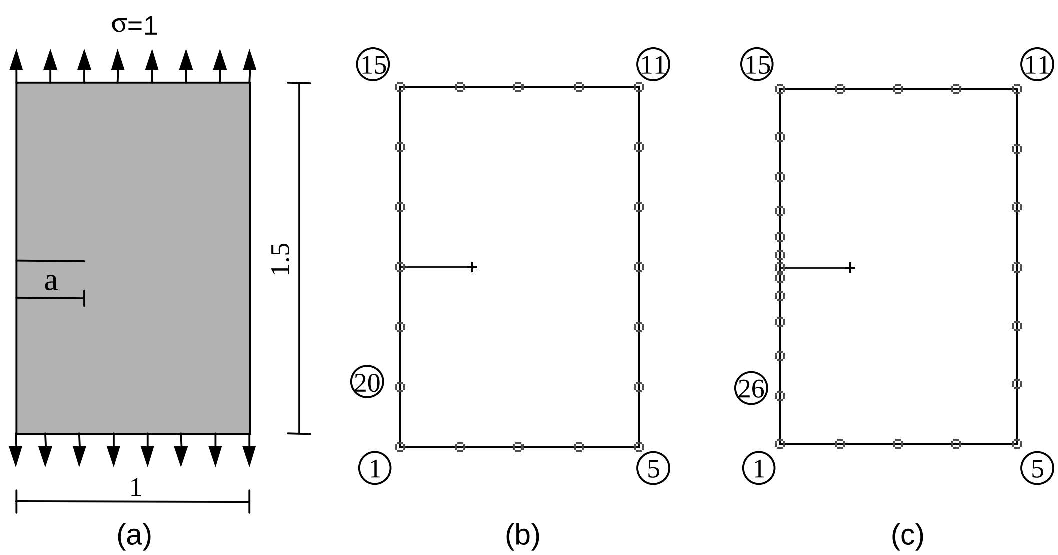

Figure 4 – Rectangular plate with an edge crack. Two boundary element discretizations are shown. Crack behavior is simulated by William's series referred to the crack tip.

Table 2 – Stress intensity factors for the edge crack of Fig. 4

Crack length	Discretization type of Fig. 4	Number of odd terms in William's series	K_I numerical	K_I analytical
0.1	b	5	0.250	0.267
0.1	c	5	0.258	0.267
0.1	c	8	0.260	0.267
0.2	b	5	0.426	0.432
0.2	b	8	0.428	0.432
0.3	b	5	0.637	0.641

6. CONCLUSIONS

Aim of this article was to investigate the possibility of modeling the cracked configuration of an elastic body in the frame of the hybrid boundary element method. The adequacy of a series of Kelvin's fundamental solutions to simulate the stress field around a crack tip was demonstrated in section 3 by considering a structure divided into sub-regions. In section 4, it was shown that hypersingular functions describe adequately the relative displacements of the crack faces. Finally, it was demonstrated how a Green's function – William's series – may be introduced in the formulation in order to improve the numerical model. At present, the authors are working on the combination of hypersingular functions and Green's functions to properly simulate the cracked behavior of an elastic body. In principle, the model should work for multiple and curved cracks. The main advantage of the proposed approach is that the stress intensity factors, as coefficients of a Green's function, are primary unknowns, whose evaluation does not require any post-processing.

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